Coupled-Oscillator Arrays for Millimeter-Wave Power-Combining and Mode-Locking

Robert A. York and Richard C. Compton†

Department of Electrical Engineering
University of California
Santa Barbara, CA 93106

†School of Electrical Engineering
Cornell University
Ithaca, NY 14853

Abstract — Arrays of coupled oscillators have recently been considered for millimeter-wave power-combining. Such systems possess a number of interesting and potentially useful nonlinear dynamical phenomena. A new theory describing arrays of coupled millimeter-wave oscillators is presented, and two important applications of such arrays—CW power combining, and a new mode-locking technique for pulse generation—will be discussed. Conditions for establishing mutual synchronization and the correct phase relationships have been investigated with the theory, and verified experimentally using several prototype X-band arrays.

1. Introduction

Obtaining useful levels of power from solid-state millimeter-wave systems will require combining the power from hundreds, or even thousands of individual active devices. Quasi-optical device arrays [1] have been suggested as an efficient solution to this problem. One quasi-optical architecture that has been successfully demonstrated is the coupled-oscillator array [2]. In this approach, individual solid-state oscillators with integrated antennas are grouped in an array. Mutual coupling between the array elements then synchronizes frequency and phase relationships, so that coherent power-combining takes place in the free-space over the array. In addition to CW power-combining, these coupled-oscillator arrays can be operated in a fundamentally new mode of operation [3–4] which produces periodic trains of high-energy pulses. This new operation is based on a mode-locking technique similar to that used in the short-pulse laser community. Furthermore, mode-locked arrays possess a beam-scanning property, which suggests their use in a radar application.

Some rudiments of a coupled-oscillator theory have been published to date [2]. This theory has been expanded to explain the recent experimental observations of mode-locked pulse trains. The theory is important in determining the physical parameters required to synthesize a particular mode of operation. New theoretical results are presented in this paper, including extensive computer simulations of array phase dynamics. The theory is rigorously verified using several Gunn diode arrays operating at X-band. In addition, new experimental results are presented for both the CW power-combining and mode-locking applications.

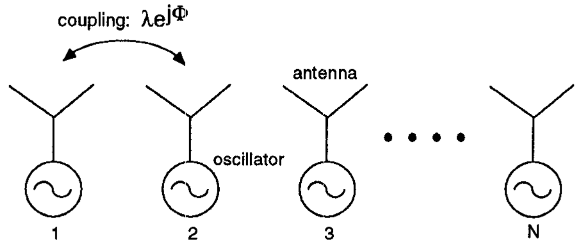

Figure 1 — Schematic showing the essential elements of a quasi-optical coupled-oscillator array. Each oscillator contains an integrated antenna, and mutual coupling between antennas leads to oscillator interaction.

2. Coupled-Oscillator Theory

The system under consideration is shown schematically in figure 1. The oscillators are modelled by a simple, single-tuned resonant circuit, with a lumped negative resistance representing the active device; this simple model has been surprisingly successful in predicting the behaviour of many types of oscillators, using a variety of devices. The mutual interaction between oscillators is assumed to be described by a complex coupling coefficient. For coupling between adjacent oscillators, this coupling parameter is written as $\lambda \exp(-\jmath\Phi)$. Following a standard method-of-averages approach [10–11], and assuming only nearest-neighbor interaction amongst array elements, yields the set of equations for N oscillators

$$\frac{dA_i}{dt} = \frac{\mu\omega_i}{2Q}(1 - A_i{}^2)A_i + \frac{\lambda\omega_i}{2Q}\sum_{\substack{j=i-1\\ j\neq i}}^{i+1} A_j \cos(\Phi + \theta_i - \theta_j)$$
$$\frac{d\theta_i}{dt} = \omega_i - \lambda\frac{\omega_i}{2Q}\sum_{\substack{j=i-1\\ j\neq i}}^{i+1} \frac{A_j}{A_i}\sin(\Phi + \theta_i - \theta_j) \qquad (1)$$

IF1

CH3141-9/92/0000-0429$01.00 © 1992 IEEE

where $i = 1, 2, \ldots, N$, and A_i, ω_i, and θ_i are the amplitude, frequency, and instantaneous phase, respectively, of oscillator i, and Q is the Q-factor of the oscillator embedding circuit. These equations are in a form of coupled Van der Pol oscillators [10]. For $\lambda = 0$ the oscillators are uncoupled, and (1) reduces to a set of isolated limit cycle oscillators with amplitudes $A_i = 1$ and frequencies ω_i. These equations are quite general, but are difficult to attack analytically because of the nonlinearities.

3. CW Power-Combining

If the coupling is not too strong, then the amplitudes of the oscillators will remain close to their free-running values, and the dynamics will be essentially contained in the phase equation alone. Furthermore, if the free-running frequencies are close enough, then the oscillators can lock to the same frequency. When this happens, $d\theta_i/dt \equiv \omega$ in the steady-state, giving

$$\omega = \omega_i \left[1 - \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{\lambda_{ij}}{2Q} \frac{A_j}{A_i} \sin(\phi_i - \phi_j + \Phi_{ij}) \right] \tag{2}$$

where $i = 1, 2, \ldots, N$. This equation can be viewed as a generalization of Adler's equation [6]. Noting that one of the phase variables is arbitrary and can be set to zero, this is a set of N equations with N unknowns (ω is an unknown), which can in principle be solved for the unknown phase distribution and steady-state synchronized frequency. In general there are many possible phase distributions which satisfy (2), but not all are necessarily stable solutions. Mode stability can be analyzed using a perturbation analysis, which leads to a linear matrix equation. This stability analysis results in an additional constraint on the phase variables.

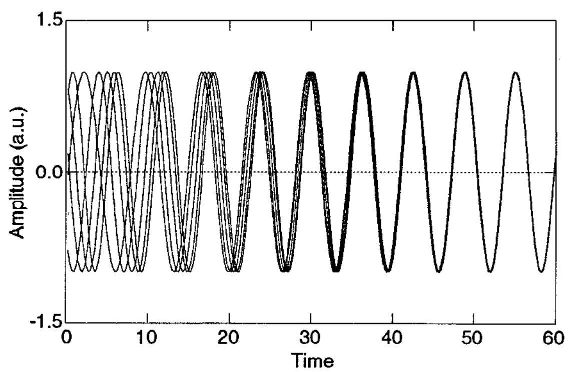

Figure 2 — Computer simulation of five nearly identical coupled oscillators. After an initial turn-on transient, the oscillators synchronize to a common frequency and phase.

Equation (2) was first considered in [2] for some simple cases, to gain insight into the operation of oscillator chains. Recently, (2) and the corresponding stability constraint have been investigated analytically and through computer simulations, in order to determine the necessary conditions for coherent power-combining. Figure 2 shows an example simulation. The theory has also been experimentally verified in a systematic and rigorous manner. Individual Gunn oscillators [5] were fabricated and mounted on small aluminum carriers, permitting a continuous variation of oscillator spacings and hence the coupling between them. Antenna patterns were then measured to characterize the phase and amplitude distribution. A comparison of theoretical and experimental radiation patterns for one particular example is shown in figure 3. The theory curve was calculated using the phase distribution found from solving (2) and the stability constraint. Excellent agreement is observed between theory and experiment regarding the number and placement of lobes and nulls in the patterns.

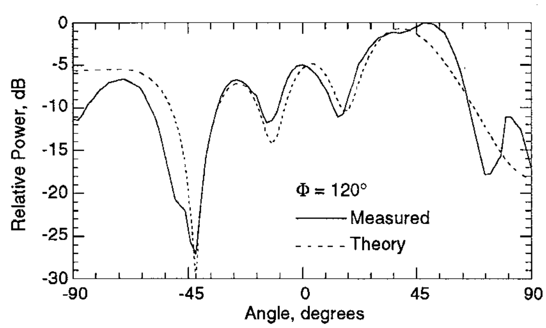

Figure 3 — Radiation patterns for four coupled oscillators, for one particular value of the coupling phase Φ. The excellent agreement verifies that the theoretical formulation adequately predicts the observed phase distribution.

4. Mode-Locked Arrays

Mode-locking refers to a situation where a number of equally spaced spectral modes are simultaneously produced and "locked" in phase, thereby producing a periodic train of pulses. This technique is widely used in short-pulse laser systems. Mathematically, the superposition of a set of $N = 2n + 1$ different spectral modes can be written as

$$E(t) = \sum_{i=-n}^{n} E_i \exp\{\jmath(\omega_i t + \phi_i)\} \tag{3}$$

The conditions for equally-spaced frequencies and locked phases are

$$\omega_i = \omega_0 - i\Delta\omega \qquad i = -n, \ldots, n \tag{4a}$$

$$\phi_i - \phi_{i-1} = \Delta\phi \tag{4b}$$

where $\Delta\omega$ and $\Delta\phi$ are constants. Assuming these conditions are met, and with equal amplitudes, $E_i = E_0$, then (1) can be written as

$$E(t) = E_0 \frac{\sin\left[N(\Delta\omega t + \Delta\phi)/2\right]}{\sin\left[(\Delta\omega t + \Delta\phi)/2\right]} \exp(\jmath\omega_0 t) \qquad (5)$$

which has the form of a carrier signal at a frequency ω_0 modulated by a periodic train of pulses, with pulse repetition frequency $\Delta\omega$.

Recent demonstrations have shown that coupled-oscillator arrays can also be operated in this mode [3–4]. In this case, each oscillator runs at a different frequency, and mutual pulling effects "lock" the system into a mode-locked state. This can be explained with reference to figure 4. When an external signal is injected into an oscillator which is outside the locking bandwidth of the the oscillator, a spectrum of frequencies due to beating effects is produced [6–8]. The additional frequencies are equally spaced by an amount proportional to the difference in the injected and free-running frequencies of the oscillator. In addition, Armand [8] has shown that there is also a constant phase progression among these additional spectral components. The idea behind the mode-locked array is to use the additional spectral components arising from the mutual pulling effects to injection-lock other oscillators in the system. In this way, the two key requirements (4) for mode-locking can be enforced.

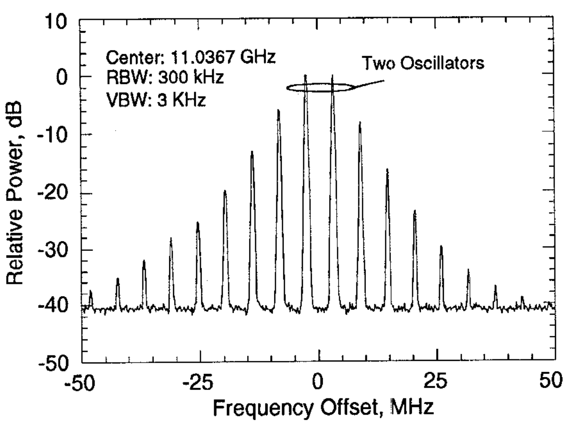

Figure 4 — Frequency spectrum of two coupled oscillators. Mutual pulling effects give rise to a number of additional spectral components. These can be used to injection-lock other oscillators.

Figure 5 shows the measured output power (with carrier removed) of a five-element mode-locked array. This measurement was made using a high-speed sampling oscilloscope, with the 11 GHz carrier removed using an envelope detection feature. Also shown for comparison is the theoretical expression (5) for five equal amplitude oscillators, with good agreement observed between the theory and measurement. As the number of modes (or in this case, oscillators) increases, the pulse width narrows and the peak power increases.

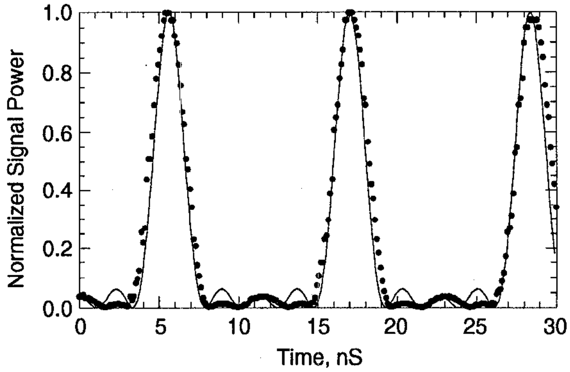

Figure 5 — Measured (dots) and theoretical (solid line) time dependence of the output signal power envelope of five mode-locked oscillators.

When the individual oscillators are spatially separated at distances comparable to a wavelength (at the carrier frequency), it can be shown that the system becomes a continuous scanning array. The total electric field above the array can then be written as

$$E(r,\theta,t) = \sum_{i=-n}^{+n} E_i G(\theta) \exp\left\{\jmath\left[(\omega_i t + \phi_i + i k_0 \Delta l\right]\right\} \qquad (6)$$

where k_0 is the free-space propagation constant, $G(\theta)$ is the amplitude gain function of each antenna element, and Δl is a path difference given by $\Delta l = \Delta d \sin\theta$. Assuming the conditions for mode-locking (4) and equal amplitudes gives the following expression

$$E(r,\theta,t) = G(\theta) \frac{\sin\left[N(\Delta\omega t + k_0 \Delta d \sin\theta)/2\right]}{\sin\left[(\Delta\omega t + k_0 \Delta d \sin\theta)/2\right]} e^{\jmath\omega_0 t} \qquad (7)$$

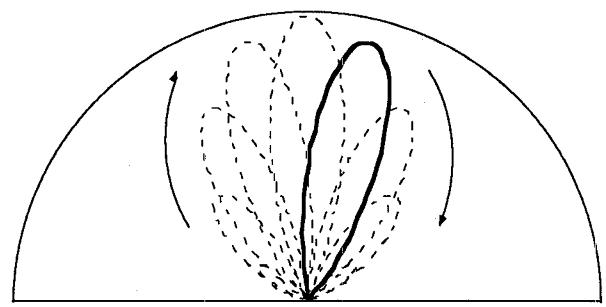

Figure 6 — Polar antenna plot simulating pulse scanning for a five-element mode-locked array, using patch antennas. The elements are spaced one-half wavelength apart. Only the main lobes have been drawn for clarity, at equally spaced time increments over one cycle.

Equation (7) has been plotted in figure 6 for a five-oscillator patch antenna array, at several time increments during one cycle of the pulse train. In this figure, a simple model for patch gain function has been assumed [9] and the element spacing is $\lambda_0/2$ where λ_0 is the free-space wavelength. The pulse repetition frequency is 100 MHz. Note from (7) that the amount of scan coverage is determined by the element spacing Δd and the gain function $G(\theta)$. If all the oscillators were located at a single point ($\Delta d = 0$), there would be no beam scanning.

5. Conclusions

Systems of coupled oscillators possess a number of interesting and useful nonlinear dynamical phenomena. Arrays of coupled oscillators have been used to model complicated biological and neural activity [12], and have now proved useful in power-combining applications. For narrow distributions of natural (or free-running) frequencies, all oscillators can synchronized to a common frequency through the phenomenon of injection locking. This is the required mode for CW power-combining. When the frequencies are very widely distributed, this mutual synchronization is impossible, and the system can exhibit chaotic behaviour. However, we have found experimentally that by carefully chosing the frequency distribution, a stable mode-locked state can be established in which the collective output of the oscillator array consists of a train of pulses, similar to a mode-locked laser. When the oscillators are spatially separated, the system generates a scanned beam. A theory describing the coupled-oscillator dynamics has been developed to explore these new effects, and the theory is amply supported with empirical evidence obtained with small arrays of X-band oscillators.

6. References

[1] J. W. Mink, "Quasi-Optical Power Combining of Solid-State Millimeter-Wave Sources", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 273-279, Feb 1986.

[2] R. A. York and R. C. Compton, "Quasi-Optical Power-Combining using Mutually Synchronized Oscillator Arrays", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-39, pp. 1000-1009, June 1991.

[3] R. A. York and R. C. Compton, "Mode-Locked Oscillator Arrays", *IEEE Microwave Guided-Wave Lett.*, vol. 1, pp. 215-218, Aug 1991.

[4] R. A. York and R. C. Compton, "Experimental Observation and Simulation of Mode-Locking Phenomena in Coupled-Oscillator Arrays", to appear in *J. App. Phys.* 1992.

[5] H. J. Thomas, D. L. Fudge, and G. Morris, "Gunn Source Integrated With Microstrip Patch", *Microwaves & RF*, pp. 87-89, Feb. 1985.

[6] R. Adler, "A Study of Locking Phenomena in Oscillators", *Proc. IRE*, vol. 34, pp. 351-357, June 1946; also reprinted in *Proc. IEEE*, vol. 61, pp. 1380-1385, Oct 1973.

[7] K. Kurokawa, "Injection-Locking of Solid-State Microwave Oscillators", *Proc. IEEE*, vol. 61, no. 10, pp. 1386–1409, Oct 1973.

[8] M. Armand, "On the Output Spectrum of Unlocked Driven Oscillators", *Proc. IEEE (Lett.)*, vol. 57, pp. 798–799, May 1969.

[9] J. R. James, P. S. Hall, C. Wood, *Microstrip Antenna Theory and Design*, Peter Peregrinus Ltd., London, 1981.

[10] B. van der Pol, "The Nonlinear Theory of Electric Oscillations", *Proc. IRE*, vol. 22, pp. 1051–1085, Sept 1934.

[11] F. Verhulst, *Nonlinear Differential Equations and Dynamical Systems*, Springer: Berlin, 1990; Chapter 11.

[12] A. H. Cohen, P. J. Holmes, and R. H. Rand, "The Nature of the Coupling between Segmental Oscillators of the Lamprey Spinal Generator for Locomotion: A Mathematical Model", *J. Math. Bio.*, vol. 13, pp. 345–369, 1982.